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Mathematical modeling and bifurcation analysis of a delayed eco-epidemiological model with disease in predator and linear harvesting

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ABSTRACT

This study proposes a three-dimensional prey-predator model with disease in predators, including the time delay due to the gestation of the predator population. The positive invariance of the solutions and the existence of equilibria in the proposed system have been explained. We also analyse the local stability of all feasible equilibrium points for the delayed and non-delayed systems. Additionally, the system's Hopf bifurcation with regard to several parameters has been investigated. Further, the existence of periodic solutions through Hopf bifurcation is shown with respect to time delay, and it is also shown that time delay is crucial in order to control the dynamics of the system. Explicit formulas are presented to determine the direction of the Hopf bifurcation and the stability of the bifurcating periodic solution using the normal form theory and the centre manifold theorem. The outcome of the theoretical investigation is ultimately validated through numerical simulation.

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KEYWORDS
Prey–predator; Hopf bifurcation; gestation; time delay; stability; bifurcation analysis; numerical simulations

1. Introduction

Q1 Nearly a hundred years ago, Lotka [1] and Volterra [2] proposed and analysed a simple prey-predator model. Since then, a remarkable amount of work has already been done, and it has also been increasing year after year. Since transmissible disease in an interacting population cannot be neglected, it is imperative from both the mathematical and ecological points of view to study the eco-epidemiological model. Kermack and McKendrick [3] first studied a deterministic epidemic model, and after that study, ecoepidemic models increased rapidly. A very significant number of studies have already been done in this area; see [4–14]. Following the work of Venturino [7], Zhang and Sun [8] considered a predator-prey model with disease in the predator and were shown to provide sufficient conditions for longevity. In [9], Guo et al. considered the following eco-epidemiological model:

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\frac{mxy}{1+gx},\\ \frac{dy}{dt} &= \frac{mxy}{1+gx}-h_1y-\beta yz, \qquad (1)\\ \frac{dz}{dt} &= \beta yz-h_2z,\end{aligned}$$

where $x(t), y(t)$ and $z(t)$ represent the densities of the prey, susceptible predator, and infected predator populations at a time t, respectively. The parameters r, K, m, n, h_1, h_2, g and β are positive constants.

When studying prey-predator models, one important factor is functional response. The number of successfully attacked prey per predator as a function of prey density is the definition of functional response. A significant number of studies have been done involving different types of functional responses [15–30].

By including time delays, the models exhibit more complex behaviour and take a more practical approach to comprehending prey–predator interactions. Time delay due to gestation for predators is a common example, and this occurs when a predator waits a while before introducing young predators after eating prey, which is a more typical and realistic time delay in an ecosystem. Therefore, more realistic models of interacting populations have been studied over the years, incorporating time delays into biological models [31–33]. Many scholars proposed and analysed delayed ecoepidemic models with disease infection in the prey [11,34–37] and disease infection in the predator [38–47].

In this paper, motivated by the works of Guo et al. [9], we are concerned with the combined effects of a transmissible disease spreading in predator populations and a time delay due to the gestation of the

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